WATER RESOURCES RESEARCH, VOL. 36, NO. 10, PAGES 2835–2844, OCTOBER 2000

A simplified model for the transient water budget of a shallow unsaturated zone

Carver S. Simmons and Philip D. Meyer

Environment Technology Division, Pacific Northwest National Laboratory, Richland, Washington

Abstract. A simplified model describing the transient water budget of a shallow unsaturated soil profile is presented and applied to an example of long-term climatic conditions for a site near Coshocton, Ohio. Its conceptualization is simplified because the instantaneous redistribution of infiltrated water in a uniform soil profile is imposed. The model's formulation is a revision of that originally presented by *Kim et al*. [1996]. Runoff from a soil profile with uniform hydraulic properties is described by using an infiltration equation derived by *Salvucci* [1996] for ponded conditions. During storm periods the infiltration rate equals the rainfall intensity, unless that rate exceeds the saturated hydraulic conductivity, and then the input rate is determined by the infiltration equation for ponded conditions, with the excess applied water being accounted for as runoff. During interstorm periods, when the soil profile is drying due to evapotranspiration (ET) and drainage, the ET rate equals the potential evapotranspiration rate E_p, until the soil profile relative saturation s drops below a certain falling value of saturation S_f. When $s < S_f$, then the ET rate is linearly proportional to s as given by $(s/S_f)\ E_p$. Drainage flux from a profile equals the hydraulic conductivity under a unit hydraulic gradient. Drainage for a 50-year-long record of climate on a daily basis for the Coshocton site compared well with measurements from four lysimeters. It is shown that the effective saturated hydraulic conductivity for the profile is determined by the runoff amount over the simulation period, and the value of S_f can be set so that the predicted drainage approximates that measured by lysimeters. This determines the appropriate cumulative ET for the site consistent with the climate data. The appropriate value of S_f also depends on the presumed profile depth, which is the plant root depth over which ET is extracted.

1. Introduction

A water budget model for an unsaturated soil profile is commonly used to predict the partitioning of applied precipitation into infiltration and runoff and to determine the depletion of stored water by evapotranspiration and by drainage below a specified depth, usually the plant root zone. Such a model is useful when estimating groundwater recharge or the flux of water that may transport a dissolved contaminant into the groundwater. This paper describes a conceptually (and mathematically) simplified water budget model that uses a simplified description of subsurface hydraulics in conjunction with site climatic data and an estimate of potential evapotranspiration.

Dose assessment modeling conducted as part of U.S. Nuclear Regulatory Commission decommissioning analyses frequently requires estimates of contaminant transport from a source of residual radionuclide materials through the unsaturated zone to groundwater [*Meyer et al*., 1996]. The flux of water that may transport the radioactive materials to the groundwater (the drainage) is often a significant factor in the estimated dose from a site. As a result of temporal variability in climate and uncertainties in the soil properties and vegetation at a site, the appropriate drainage flux may be uncertain. The need for a simplified method to estimate the transient drainage and its uncertainty motivated the development of the water budget model described here.

Typically, a numerical solution of Richards' unsaturated flow equation is used to find the detailed water balance for a soil profile having well-delineated hydraulic properties [e.g., *Fayer et al*., 1992]. However, detailed data on the soil hydraulic properties and climate may not always be available to perform such an accurate simulation of a water balance. In addition, computational requirements may preclude a numerical solution in some cases. Several analytical or approximate solutions of Richards' equation have been derived [e.g., *Warrick et al*., 1990, 1991; *Sander et al*., 1991; *Smith et al*., 1993]. These analytical solutions are often limited by restrictions on the form of the soil hydraulic characteristic functions (e.g., as used by *Broadbridge and White* [1988]), or by use of an unrealistic surface evaporation boundary condition, or by a poor description of plant evapotranspiration, which acts as a sink term throughout the soil profile.

Marani et al. [1997] recently discussed hydrologic partitioning related to water balance modeling. That modeling study, however, was focused on an equilibrium description of the water balance. *Kim et al*. [1996] formulated a simplified, transient water budget model that uses the *Brooks and Corey* [1964] soil water characteristics and describes plant evapotranspiration as a sink term withdrawing water from the entire soil profile. *Kim et al*. [1996] demonstrated that water budget predictions for a variety of soils and climatic conditions compared favorably with numerical solutions of Richards' equation. In an application to a long-term lysimeter record, however, we found

This paper is not subject to U.S. copyright. Published in 2000 by the American Geophysical Union.

Paper number 2000WR900202.

that the model of *Kim et al*. [1996] underestimated the long-term evapotranspiration occurring at the site and significantly overestimated the drainage. Modifications to the model of *Kim et al*. [1996] to provide a more realistic and flexible representation of evapotranspiration are discussed below.

An alternative solution for the infiltration component of the water budget model is also presented. The infiltration equation recommended here is based on the derivation given by *Salvucci* [1996], and runoff is described by using the concept of the time compression approximation procedure, as discussed in detail by *Salvucci and Entekhabi* [1994]. Although there are issues of whether *Salvucci*'s [1996] infiltration equation is the most accurate solution of Richards' unsaturated flow equation for ponding conditions [*Parlange et al*., 1998; *Salvucci*, 1998], the modification presented below to the equations given by *Kim et al*. [1996] treat more accurately the long-term behavior of infiltration for purposes of accomplishing simplified modeling of a water budget subject to arbitrary size rainfall events.

One weakness of a simplified water budget model is that it must be supplied with effective hydraulic property values (appropriately averaged values) to make it comparable to actual sites where properties vary spatially [*Kim and Stricker*, 1996]. When hydraulic properties vary at a site, it is not apparent which parameter values to select to accurately model the water budget in a deterministic way. Moreover, the directly measured hydraulic properties for a particular site may not immediately provide the most appropriate parameter values to employ in a simplified water budget model. The appropriate effective model parameters may be best determined by matching the long-term water budget predictions with measurements of water budget components, especially by matching the drainage, when actual evapotranspiration is not available. The estimation of the appropriate effective parameters for the simplified model described below is demonstrated for a specific site (Coshocton, Ohio).

Kim et al. [1997] considered the issue of finding effective parameters for representing the average of an ensemble of predicted water budget realizations, as associated with a distributed range of parameters characterizing the heterogeneity. That is, they were interested in determining the effective parameters for modeling the average predicted water budget. In contrast, this paper is concerned with estimation of effective parameters for modeling the actual water balance measurements of a specific site. A lysimeter installation near Coshocton, Ohio [*Mustonen and McGuinness*, 1967], is used to confirm that measurements of drainage are consistent with the concepts of the simplified model presented below. *Bonta and Muller* [1999] also examined groundwater recharge at the Coshocton location using an empirical water budget model, which did not include any physical description of the unsaturated flow processes.

2. Water Budget Model Formulation

The simplified water budget model presented here is a modification of that derived originally by *Kim et al*. [1996]. Following the development of *Kim et al*. [1996], the water budget calculation is divided into two periods: storms with rainfall and interstorm periods with evapotranspiration and drainage. Like the model of *Kim et al*. [1996], the following conceptual simplifications are imposed: (1) a soil profile with uniform hydraulic properties; (2) no drainage during storm periods; (3) instantaneous redistribution of prior infiltrated water; (4) uniform water extraction over the profile depth due to evapotranspiration; and (5) drainage under an imposed unit hydraulic gradient.

2.1. Solution for Storm Periods

In the model of *Kim et al*. [1996] the infiltration rate before ponding occurs is equal to the rainfall intensity P:

$$i(t) = P. \tag{1}$$

Equation (1) applies up to a ponding time t_p, after which the infiltration is determined by a constant head boundary condition at the soil surface. *Kim et al*. [1996] used the solution of *Philip* [1957] for the infiltration occurring after ponding for an initial saturation s_o:

$$i_h(t) = \tfrac{1}{2}S(s_o)t^{-1/2} + ak_s, \tag{2}$$

where $S(s_o)$ is the sorptivity and k_s is the saturated value of hydraulic conductivity. The constant, a, is less than one, rendering (2) invalid at large times.

Using the *Brooks and Corey* [1964] relationships to describe the soil water retention and hydraulic conductivity,

$$s(\psi) = \left(\frac{\psi}{\psi_s}\right)^{-m} \qquad \psi < \psi_s$$
$$s = 1 \qquad \psi \geq \psi_s \tag{3}$$

$$k(s) = k_s(s)^{(2+3m)/m}. \tag{4}$$

Kim et al. [1996] derived an expression for the sorptivity (squared):

$$S(s_o)^2 = \left[\frac{2\theta_s(1 - s_o)\psi_s}{1 + 3m}\left[s_o^{(1+3m)/m} - 1\right]\right]k_s. \tag{5}$$

In these expressions, ψ is the matric potential, ψ_s is the air entry value of matric potential, m is the pore size distribution index, and θ_s is the saturated water content.

In the present water budget model the approximate infiltration equation of *Salvucci* [1996] is used in place of (2). *Salvucci*'s [1996, equation (37)] infiltration equation under the same ponded boundary condition is

$$i_h(t) = (k_s - k_o)f(\sqrt{\tau}) + k_o \qquad \tau = \frac{t}{t + \chi}, \tag{6}$$

where k_o is the hydraulic conductivity at $s = s_o$, the function f is defined as

$$f(\xi) = (1 - \xi^2)^2\left[\frac{\sqrt{2}}{2}\frac{1}{\xi} + \frac{(2 - \xi)}{4(1 - \xi)^2}\right], \tag{7}$$

and χ is a scaling factor for time as defined by *Salvucci* [1996, equation (10)]:

$$\chi = \frac{1}{2}\left(\frac{S(s_o)}{k_s - k_o}\right)^2. \tag{8}$$

(Here the function (7) does not have the same definition as originally defined by Salvucci; it is instead the normalized dimensionless infiltration rate.) *Salvucci* [1996, equation (19)] achieved a more general solution for infiltration by introducing the scaled time variable τ, as defined here in (6). A reader should note that although (6) is an approximation, it nevertheless avoids the dependence on a and is valid for an indefinite time.

According to *Salvucci* [1996, equation (38)], the cumulative infiltration is

$$i^{\text{cum}}(t) = F(\sqrt{\tau}), \tag{9}$$

where

$$F(\xi) = \left[(k_s - k_o)\left[\sqrt{2}\,\xi + \frac{\xi^2}{2(1-\xi)} \right] + \frac{k_o}{\left(\frac{1}{\xi^2} - 1 \right)} \right] \chi. \tag{10}$$

The time compression approximation procedure as described by *Salvucci and Entekhabi* [1994] and as applied by *Kim et al.* [1996] is used to match (1) and (6). The equivalent ponding time t_e occurs when the infiltration rate given by (6) just equals the rainfall rate P. That time is thus determined by solving the following:

$$\frac{P - k_o}{k_s - k_o} = f(\xi_e) \tag{11}$$

for

$$t_e = \frac{\chi}{\left(\frac{1}{\xi_e^2} - 1 \right)}. \tag{12}$$

The actual ponding time is now derived, by applying the condition given by *Kim et al.* [1996, equation (12)]. It is obtained by equating the cumulative infiltration up to the equivalent ponding time t_e to that produced by the steady rainfall rate P up to the ponding time t_p, which gives

$$t_p = \frac{F(\xi_e)}{P}. \tag{13}$$

After the ponding time is reached, the cumulative infiltration is

$$i^{\text{cum}}(t) = Pt_p + I(t - t_p + t_e) - I(t_e) \qquad t > t_p, \tag{14}$$

with

$$I(t) = F\left(\sqrt{\frac{t}{t + \chi}} \right). \tag{15}$$

The following equations from *Kim et al.* [1996] are used with (14) to calculate the total cumulative infiltration, cumulative infiltration excess, and profile saturation during a storm period:

$$i^{\text{cum}}(t) = Pt \qquad t \le t_p \tag{16}$$

$$q_{ie}^{\text{cum}}(t) = Pt - i^{\text{cum}}(t) \tag{17}$$

$$s(t) = s_o + \frac{i^{\text{cum}}(t)}{d_r \theta_s}, \tag{18}$$

where d_r is the profile rooting depth. If $s(t)$ of (18) exceeds unity, it is set to 1, and the infiltration amount producing the saturation greater than unity is accumulated in the water budget as "saturation excess runoff," following the definition of *Kim et al.* [1996, equation (19)].

Note that the infiltration models as formulated here and by *Kim et al.* [1996] are actually schemes to estimate runoff, because the infiltration is just equal to the rainfall amount, unless the ponding time is reached during the rainfall duration. The infiltration equations under ponded conditions are then used to determine what part of the rainfall is rejected after the soil surface can no longer sustain the water input rate imposed by the rainfall rate P.

Infiltration is always greater for Salvucci's model than for Philip's model. As a consequence, the Salvucci model always produces less runoff. The difference between the two models increases as the duration of the rainfall increases. Not only does the Salvucci infiltration equation yield a solution that is expected to remain more accurate over longer time [*Salvucci*, 1996], but it also does not rely on the parameter a as used by *Kim et al.* [1996]. This parameter, a, is ambiguous if the infiltration is sufficiently long or lasts for an unknown duration.

2.2. Solution for Interstorm Periods

The evapotranspiration model of *Kim et al.* [1996] was restricted to a linear relation between the evapotranspiration rate and saturation. This made possible deriving an analytical expression for the soil water depletion, but it does not provide flexibility so that predicted evaporation and drainage conform to site-specific climatic conditions. The evapotranspiration model used here introduces a saturation falling value S_f so that the potential evapotranspiration rate E_p is maintained until the soil saturation is less than S_f. The evapotranspiration rate function is

$$\begin{aligned} E(s) &= \left(\frac{s}{S_f} \right)^p E_p \qquad s < S_f \\ E &= E_p \qquad s > S_f. \end{aligned} \tag{19}$$

The p exponent controls the rate at which evapotranspiration decreases for s below S_f and is included to make the relationship more general. This model of evapotranspiration is similar to that presented by *Feddes et al.* [1976].

Following the development of *Kim et al.* [1996], the profile saturation during an interstorm period is represented by

$$ds/dt = -(q_p + E)/(d_r \theta_s), \tag{20}$$

where q_p is the drainage flux, which equals (4) under the imposed unit gradient drainage condition. Using (4) and (19), (20) can be expressed in terms of a scaled time T as

$$\frac{d}{dT} s = (-1)\left(s^C + \frac{E(s)}{k_s} \right), \tag{21}$$

where $C = 3 + 2/m$ and with

$$T = \frac{t}{T_{\text{scale}}} \qquad T_{\text{scale}} = \frac{d_r \theta_s}{k_s}.$$

As pointed out by *Kim et al.* [1996], (21) cannot be integrated analytically unless p and S_f are both equal to unity. However, in general, a simple integration will determine the solution $T(s)$. In particular, after solving for the value of s satisfying the following integral for a given T, $E(s)$ is defined by the following equation as a function of T:

$$T = \int_s^{s_o} \frac{1}{s^C + \bar{E}(s)}\, ds, \tag{22}$$

where $\bar{E}(s) = E(s)/k_s$ defines a scaled evapotranspiration rate function. Using (22) to determine s, given a value of T, then, gives the cumulative evapotranspiration as

Table 1. Base Case Profile Parameters for Coshocton, Ohio, Lysimeter Site

Parameter	Value
Saturated water content of profile	0.43
Air entry pressure, cm	−35.3
Saturated hydraulic conductivity, cm/h	1.188
Brooks-Corey pore size index m	0.653
Profile depth, cm	150
Initial profile saturation	0.25
Saturation falling value for evapotranspiration ($p = 1$)	0.233 (= 0.1/0.43)

$$E^{\mathrm{cum}}(s) = d_r\theta_s \int_s^{s_o} \frac{\bar{E}(s)}{s^C + \bar{E}(s)}\, ds. \tag{23}$$

An initial saturation s_o is reduced to $s(t)$ by the evapotranspiration according to (22) and (23).

Having determined the cumulative evapotranspiration up to time t by using (22) and (23), the cumulative drainage plus the cumulative evapotranspiration equals the change in profile water storage over the specified interstorm period. That is, the cumulative drainage during an interstorm period of time t is [*Kim et al*., 1996, equation (34)]

$$q_p^{\mathrm{cum}}(t) = d_r\theta_s(s_o - s(t)) - E^{\mathrm{cum}}(t). \tag{24}$$

Evaluating (23) and (24) is no more computationally intensive than evaluating the power series solution given by *Kim et al*. [1996, equation (31)] for the special case $S_f = 1$ and $p = 1$.

The cumulative evapotranspiration will generally increase for smaller values of S_f because the evapotranspiration rate remains equal to the potential value over a greater range of saturations. The general evapotranspiration model used here thus tends to result in a lesser cumulative drainage than the model of *Kim et al*. [1996], which is limited to the special case of $S_f = 1$ and $p = 1$, as noted previously.

3. Water Budget Simulations for an Example Site

A number of water budget simulations were carried out to illustrate the behavior of the model. These simulations used data from a lysimeter facility located at the North Appalachian Experimental Watershed, near Coshocton, Ohio, operated by the Agricultural Research Service of the U.S. Department of Agriculture [*Harrold and Dreibelbis*, 1958, 1967]. The Y101 site at this facility contains four lysimeters constructed in situ around undisturbed 1.9-m-wide × 4.27-m-long × 2.44-m-deep soil monoliths. A continuous record of runoff and drainage is available from 1944 to the present at all four lysimeters. One of the lysimeters is also equipped with a self-recording weighing mechanism. The lysimeter soils are described as silt loam grading into loam with sandstone fragments at deeper depths, eventually grading into decomposed sandstone at the base of the monoliths. The surface slope of the lysimeters is 23%. Various vegetation types and tilling practices were used on the lysimeters since their construction.

A 50-year record of daily precipitation and potential evapotranspiration was used in the water budget simulations. Potential evapotranspiration was estimated using the method of Penman [*Shuttleworth*, 1993] from measurements made at a weather station located on the Y101 site. Daily precipitation was derived from the Y101 weighing lysimeter, which was found to be more accurate than the weather station rain gage. While potential evapotranspiration was assumed to be constant over any given 24-hour period, the daily precipitation was assigned a random duration between 1 and 23 hours for those days on which precipitation was recorded. The probability distribution of durations was based on an estimate of the average number of exceedances per year for the specific daily rainfall depth observed. Exceedance values were estimated using the method of *Chen* [1983]. Thus the rainfall intensity sequence for the long term is not the actual, but it does represent the actual total rainfall on each day. Rainfall intensity is constant over the randomly selected duration. (Note that the same duration sequence was used in all simulations discussed below.) Rainfall occurring when the average daily temperature was less than 0°C (freezing point) was accumulated as snow depth. Also, no evapotranspiration was allowed on those snow days.

Empirical relationships are used for estimating snowmelt due to vapor condensation, convection, radiation, and rainfall as described by *Streile et al*. [1996]. Melting of the snow depth (equal to equivalent water depth) occurs when the average temperature is above freezing. Melted snow and rainfall are added together as water input to the storm model, by which water input exceeding the allowed infiltration then contributes to the cumulative runoff for the water budget period. Potential evapotranspiration (ET) is zero for the day as long as a snow depth is present. On a day when the entire snow depth is melted, E_p is reduced by the energy required for snowmelt. Note that the snowmelt model does not alter the storm or interstorm models' formulation but only contributes additional water for potential infiltration. For the Coshocton site example, however, snowmelt does not contribute a substantial amount of water input; therefore the snowmelt modeling is not described in any further detail. Average precipitation was 106 cm, while the average maximum snow depth (water equivalent) was 7.3 cm over the 50 years.

Soil hydraulic properties for the lysimeters were estimated from the physical description of the soils and from soil properties measured on samples from a nearby location as reported by *Kelley et al*. [1975]. No specific hydraulic property measurements from the site were available. The selected parameter values used with the water budget model are given in Table 1. These parameter values are hypothetical and at best only an initial guess for the example site. (Note that S_f, which is a parameter introduced by the ET model, is not directly measurable and has to be determined by model calibration for the site's water budget, as will be discussed below.) Water budget simulations were conducted for the 50-year period from 1945 to 1994, with the water budget estimated on an hourly timescale (i.e., the shortest-duration storm or interstorm period was 1 hour).

3.1. Lysimeter Comparison

Cumulative runoff and drainage over the 50-year period of simulation for the four Y101 lysimeters are given in Table 2. Variation between lysimeters is substantial, primarily due to the different tillage practices and vegetation grown on each of the lysimeters. Runoff on the lysimeters was relatively small, ranging from 0.5 to 2.4% of the precipitation. (Cumulative precipitation at the site was 5323 cm over the 50-year period.) Drainage varied from 25 to 37% of the precipitation. The average lysimeter values are also given in Table 2 along with the water budget model results for the base case simulation. While the simulated cumulative drainage fell within the range

of values observed at the lysimeters, the runoff result for the model was much lower than at any of the lysimeters. Recall that the base case parameter values (Table 1) were selected based on a physical description of the lysimeters and did not involve any calibration, except for S_f, which was estimated by performing preliminary simulations using the other base case parameter values.

Figure 1 shows the annual drainage behavior over the 50-year period. Only the upper and lower bounds (mean drainage plus and minus the average absolute deviation from the mean for each year) are shown for the lysimeters. The base case simulation result (solid curve) is shown for comparison. It appears that the simulation is reasonably consistent with the envelope of observed annual values; the general trends in observed drainage are followed, although the simulated drainage value is sometimes outside the measured lysimeter variation in various years. It will be shown that selecting an appropriate value of S_f allowed the predicted drainage to be matched with the lysimeter measurements.

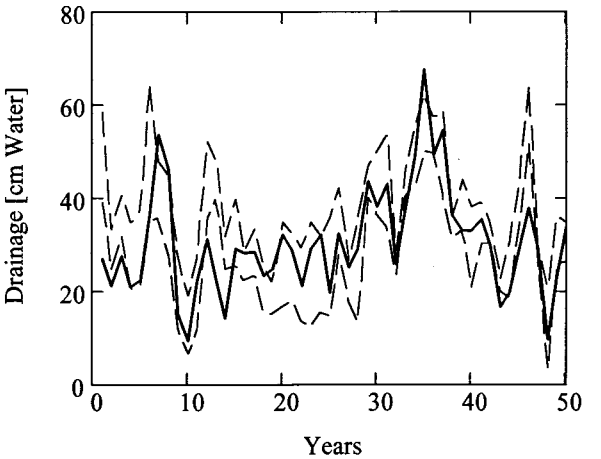

Figure 1. Model-predicted drainage in the 150-cm depth profile (solid curve) compared with four lysimeters at Coshocton, Ohio. Dashed curves indicate the envelope obtained from the measured mean drainage by taking plus and minus the averaged absolute deviation from the mean drainage of the four lysimeter measurements for each year. Model parameters are those given in Table 1. The model-predicted total drainage over 50 years is 1528 cm, while runoff is 6.7 cm.

3.2. Interstorm Comparison

To examine the interstorm model's dependence on S_f only, the storm model was restricted to the infiltration solution of *Philip* [1957], as originally used by *Kim et al.* [1996]. That is, the prediction of the revised interstorm model was tested while keeping the original storm model. Results from using values of $S_f = 1$ and $S_f = 0.233$ (the base case value) were compared. Using a value of $S_f = 1$ is equivalent to the model of *Kim et al.* [1996]. (This was confirmed by duplicating the results obtained using the original analytical solution of *Kim et al.* [1996].) The other choice of S_f, taken as less than unity, is shown later to yield an appropriate drainage amount over the simulation period. Its value is guessed based on the results of a few water budget simulations while holding fixed the other base case parameters.

The components of the annual water budget for the two values of the falling saturation are shown in Figure 2. The annual infiltration is unaffected by the value of S_f (Figure 2a), but the saturation, evapotranspiration, and drainage are significantly different. Saturation (which is the profile saturation at the end of the year) is consistently less for the lower value of S_f and tends to vary over a greater range (Figure 2b). The lower value of S_f results in a substantially larger annual evapotranspiration (Figure 2c), and it results in a correspondingly smaller annual drainage (Figure 2d). Table 3 compares the long-term water balance totals. Initial profile saturation is 0.25 for each case.

Figure 2 shows that using the revised evapotranspiration model allowed the long-term evapotranspiration (and drainage) to have an adjustable outcome. This additional flexibility in the water budget model is achieved by introducing the single parameter S_f. The model of *Kim et al.* [1996] is restricted to a value of $S_f = 1$, and as a consequence, it can produce only a single water budget for a given sequence of rainfall durations.

Table 2. Cumulative Runoff and Drainage for the Coshocton Y101 Lysimeters From 1945 to 1994 and for the Base Case Water Budget Simulation

	Runoff, cm	Drainage, cm
Lysimeter A	71	1983
Lysimeter B	129	1402
Lysimeter C	80	1803
Lysimeter D	29	1345
Lysimeter average	77	1633
Base case simulation results	7	1528

3.3. Storm (Runoff) Comparison

As discussed above, the runoff estimated with the base case parameters was substantially less than the observed runoff at the four lysimeters. To increase the cumulative runoff to a value more consistent with the lysimeter observations, the saturated hydraulic conductivity was reduced to 0.64 cm/h (from the base case value of 1.188 cm/h). This resulted in a cumulative runoff for the 50-year period of 70.4 cm while still using the *Philip* [1957] infiltration equation for the storm model. Later (in the next section, about effective model parameters; Figure 5) it is shown that the appropriate value of saturated hydraulic conductivity can be calibrated to the site runoff, with relative insensitivity to the value of S_f.

Using this lower saturated hydraulic conductivity value of 0.64 cm/h, the 50-year simulation was repeated using the *Salvucci* [1996] infiltration equation. Annual runoff for the two infiltration models is shown in Figure 3. The water budget model using the Salvucci infiltration equation yields about 13 cm less runoff and 10 cm more drainage over the 50 years. Cumulative water budget components are given in Table 4. Because runoff is relatively small for this example, the differences between the two infiltration models are minor. As shown by Table 5, the difference in runoff for the two storm models is within variations produced by using other rainfall duration sequences. For other combinations of soil type and climate, the differences, however, may be significant, particularly for intense rainfall events of longer duration.

4. Effective Model Parameters

The base case soil hydraulic parameters (Table 1) were selected as representative of the silt loam soil type contained in the lysimeters at Coshocton, except for the saturation falling

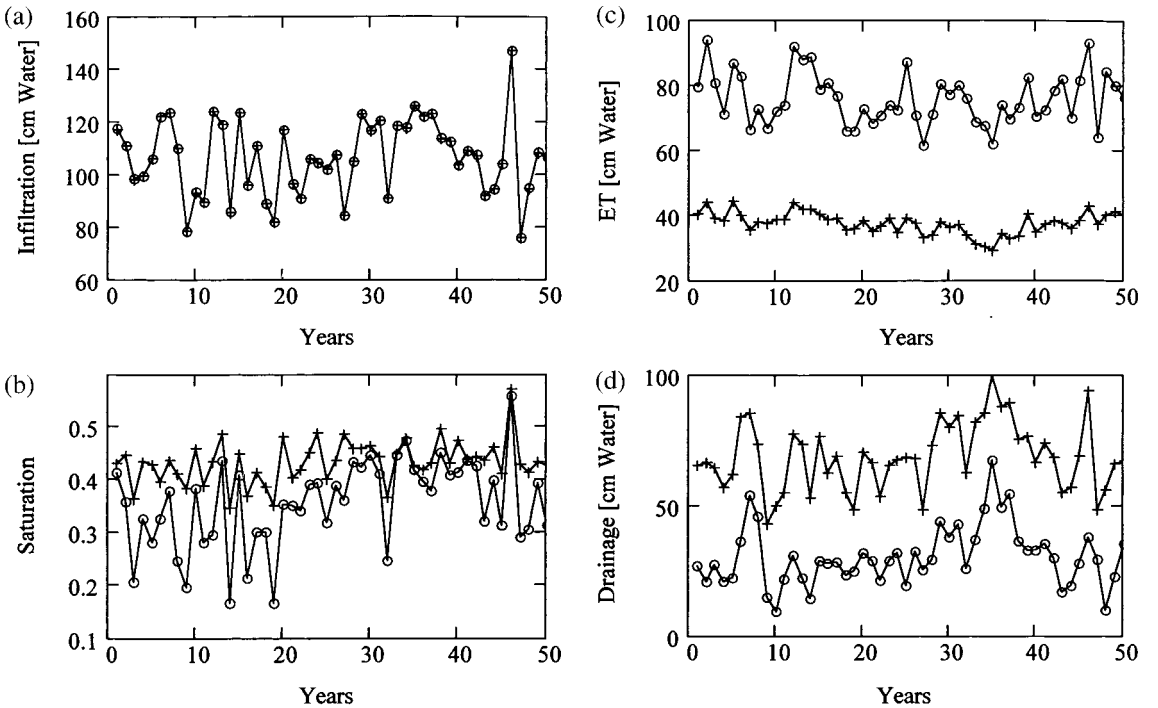

Figure 2. Components of the water balance simulated for Coshocton, Ohio. Plus sign indicate the original model of *Kim et al.* [1996] using $S_f = 1$, and circles indicate the modified model using $S_f = 0.233$. The base case parameters in Table 1 are used.

value S_f, which is established by model calibration. However, there is no reason to expect that these parameter values would accurately represent the transient water balance in a profile of arbitrary depth. For Coshocton, a 150-cm depth was chosen to describe the upper portion of the lysimeters, which were intended to be physical models of the near surface. A single value for each soil hydraulic parameter is used to describe the entire 150-cm profile in this simplified water budget model. At best these are only effective parameter values. To make the best use of the simplified water budget model, it is necessary to identify the particular effective parameter values that allow the model to mimic the actual site's water balance for the selected time period.

To help identify appropriate effective parameter values, a range of cumulative water budget components (evapotranspiration, drainage, and runoff) is calculated for a set of parameter values presumed to encompass the prediction range. Such a set of calculations is shown in Figure 4. Each point is the result of a water budget calculation for various values of the saturation falling value S_f and the saturated hydraulic conductivity K_s, using the 50-year climate record for the Coshocton lysimeter site. Resulting cumulative drainage over the 50-year period is shown as a function of the saturation falling value in Figure 4, for various values of K_s. (Note that small changes of the hydraulic parameters other than K_s are anticipated to have a minor affect on the relationship between drainage and S_f.) The curves in Figure 4 depend on two different sets of m, ψ_s, and θ_s to demonstrate the insensitivity to these hydraulic parameters, other than K_s. The initial profile saturation is always 0.25 for each simulation. The same rainfall duration sequence for the 50-year-long climate is used to produce the simulations

Table 3. Cumulative Water Budget Components for 50-Year Simulation of Coshocton, Ohio, Lysimeter Site (Philip Infiltration Model)

	Saturation Falling Value	
	$S_f = 0.233$	$S_f = 1$
Total infiltration, cm	5311	5306
Total evapotranspiration, cm	3788	1873
Total profile drainage, cm	1519	3422
Total runoff, cm	11.7	16.9
Storage change, cm	4.0	10.7

Total water input = infiltration + runoff. Total infiltration = evapotranspiration + drainage + storage change.

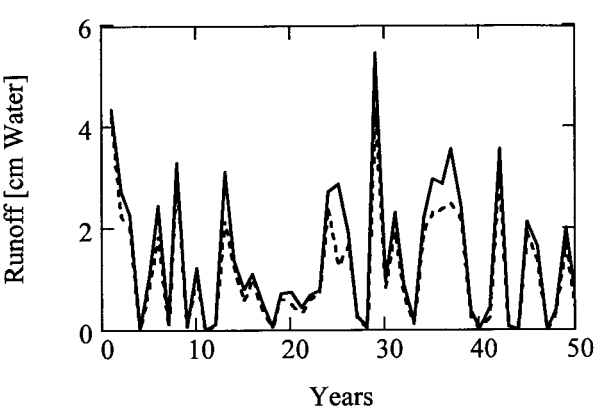

Figure 3. Runoff by the Philip (solid curve) and Salvucci (dashed curve) infiltration models. Base case parameters are used, except with $K_s = 0.64$ cm/h instead. Water budget cumulatives are given in Table 4.

with the expectation that the relative relationships would be maintained for any other duration sequence, although the particular predicted values of drainage and runoff would change (e.g., Table 5).

Cumulative drainage is thus highly sensitive to the value of S_f. Recall that $S_f = 1$ (here with $p = 1$) corresponds to the original model of *Kim et al.* [1996], whereas $S_f = 0$ is the case where evapotranspiration always equals the maximum. The curves in Figure 4 differ mainly as a consequence of the different K_s values; drainage tends to decrease as K_s is decreased, for a fixed value of S_f. The parameters other than S_f and K_s are primarily held at their base case values. Except in one instance (indicated by the squares in Figure 4), the pore size index m, the air entry pressure ψ_s, and the saturated water content θ_s deviate from the base case values. The similar trend of the curves in Figure 4 suggests that the drainage is not very sensitive to m or ψ_s over the limited range of values considered, although this is not an exhaustive study of sensitivity to those parameters for this example. *Kim et al.* [1996] have discussed in more detail the sensitivity of the model to the parameters m and ψ_s. The range of drainage (1000–3500 cm of water) that is possible as shown in Figure 4 also encompasses the observations for the lysimeters at Coshocton (Table 2). Thus, on the basis of Figure 4, $S_f = 0.233$ is a reasonable choice to match the expected range of lysimeter drainage. However, that value is not intended in any sense to give an optimal correspondence with the actual average drainage.

The reason for the trend as seen in Figure 4 is explained as follows. The total drainage in Figure 4 is always linearly related to cumulative evapotranspiration so that the sum of these budget components is approximately equal to the cumulative infiltration over the simulation period. Hence an increase in evapotranspiration implies a corresponding decrease in drainage amount. Thus, when S_f is decreased, the cumulative evapotranspiration will be increased, because $E = E_p$ is extracted for a longer time during each interstorm period. This increased evapotranspiration then produces a decreased total drainage for a fixed amount of infiltration. Therefore drainage decreases when S_f is decreased, while all other hydraulic parameters are held fixed.

It is readily discovered that the infiltration model used here is essentially a description of the runoff response. In particular, the infiltration amount is usually equal to the water input, unless the storm event is intense enough so that ponding occurs for the particular rainfall and initial moisture condition. The initial moisture condition for each infiltration event depends, of course, on the prior history of the water application. Thus, for a particular fixed climatic input, the runoff is going to be a direct reflection of the profile saturated hydraulic conductivity. In more realistic situations the runoff would be mostly a consequence of the surface soil conditions only. Although this aspect is a conceptual limitation of this simplified model, it nevertheless allows for the possibility that the appropriate value of saturated hydraulic conductivity can be identified so that runoff is consistently estimated for the site.

Figure 5 shows that cumulative runoff depends mainly on the saturated hydraulic conductivity for a fixed climatic description (a particular rainfall duration sequence). Notice that the curves for different values of S_f, which would produce quite different estimates for the total drainage and evaporation, are not greatly separated. That is, the runoff amount is nearly the same, depending mainly on K_s, regardless of the S_f value. Thus Figures 4 and 5 together suggest that it is nearly possible to determine S_f from the drainage and K_s from the runoff, as if those predicted water budget components were independently determined. (However, as shown in Figure 6, runoff is not actually independent of S_f, but is mainly determined by K_s. Also, note that runoff is not insensitive to a combined change in m and ψ_s. In particular, for the lowest value of K_s indicated in Figure 5 and for $S_f = 0.233$, when m, ψ_s, and θ_s take the values other than base case as in Figure 4, the runoff is about 90 compared with 250-cm water as indi-

Table 4. Cumulative Water Budget Components for 50-Year Simulation of Coshocton, Ohio, Lysimeter Site Using Philip and Salvucci Infiltration Equations

	Infiltration Model	
	Philip [*Kim et al.*, 1996]	*Salvucci* [1996]
Total infiltration, cm	5253	5266
Total evapotranspiration, cm	3889	3893
Total drainage, cm	1359	1369
Total runoff, cm	70.4	57.0
Storage change, cm	4.4	4.5

Base case parameters are used except that $K_s = 0.64$ cm/h.

Table 5. Cumulative Water Budget Components for 50-Year Simulation of Coshocton, Ohio, Lysimeter Site Using Three Rainfall Duration Sequences (Philip Infiltration Model)

	Sequence 1	Sequence 2	Sequence 3
Total infiltration, cm	5253	5265	5284
Total evapotranspiration, cm	3889	3887	3895
Total drainage, cm	1359	1374	1383
Total runoff, cm	70.4	57.7	39.4

$K_s = 0.64$ cm/h, $m = 0.653$, $\psi = -35.3$ cm, $S_f = 0.233$, and depth = 150 cm.

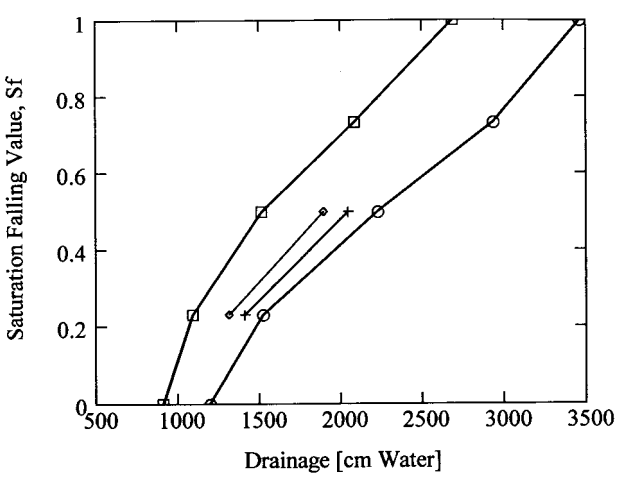

Figure 4. Drainage depending on falling value of saturation of evapotranspiration model. Parameters for each water balance simulation are $K_s = 0.336$ cm/h, $m = 0.414$, $\psi_s = -70.3$ cm, $\theta_s = 0.405$ (squares); base case $K_s = 1.188$ cm/h, $m = 0.653$, $\psi_s = -35.3$, $\theta_s = 0.43$ (circles); and $K_s = 0.549$ cm/h (diamonds) and $K_s = 0.762$ cm/h (plus signs), both using the remaining base case parameter values.

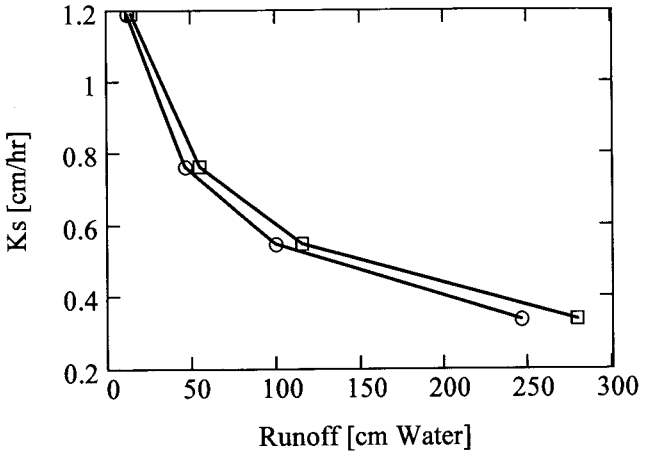

Figure 5. Runoff depending on saturated hydraulic conductivity. Falling value of saturation is $S_f = 0.233$ (circles) and $S_f = 0.5$ (squares). Other parameters have the base case values (Table 1).

cated in Figure 5. Figure 6 gives the runoff and drainage for these other parameter values and indicates that those water budget components are more dependent as K_s decreases from the base case value.)

Figure 5 indicates that the value of $K_s = 0.64$ cm/h is more appropriate than the base case value to produce the correct value of runoff consistent with the Coshocton lysimeter measurements (Table 2). Note that this chosen value of K_s is not intended to yield the average runoff exactly; it only gives a runoff value within the lysimeter measurement range.

The prediction of runoff and drainage for this selected value of K_s is shown in Figure 6 (as the larger open square), compared with the actual measurements (the four crosses). As seen earlier, the base case value of $K_s = 1.188$ cm/h gives too small a runoff (lowermost curve) when compared with the actual measurements. Also, the other values, $K_s = 0.336$ cm/h, $m = 0.414$ (Figure 4) with $S_f = 0.5$, can yield drainage and runoff consistent with the Coshocton lysimeters. Notice that the two line segments for $K_s = 0.549$ and 0.762 cm/h for $S_f = 0.233$ and 0.5 bracket the zone of the runoff and drainage where the water budget prediction (large square) falls. Apparently, the identified effective K_s would have to be decreased below $K_s = 0.336$ cm/h (large square moved upward) to attain the greatest measured runoff, whereas K_s would have to be between 0.762 and 1.188 cm/h (large square moved downward) to attain the lowest measured runoff. Thus the suitable model parameters can be identified to represent the Coshocton site water budget (for either of the four lysimeters), but the effective parameter combinations would not be unique. However, the actual measurements for the Coshocton lysimeters are by no means unique either.

The predictions given in Figures 4–6 depend on the particular fixed sequence of rainfall durations; changing that sequence would alter the curves in the figures. However, the relational dependence of the water budget model's predictions on its parameters would not be altered by a different realization of rainfall durations. To demonstrate the influence of the rainfall duration sequence, Table 5 gives the water balances for three realizations of duration sequence, including that used for the figures. The model parameters are those selected to best represent the Coshocton water budget (i.e., the base case values now with $K_s = 0.64$ cm/h, as indicated by the large square in Figure 6). The total applied water is about 5323 cm for each simulation. The duration sequences (realizations of storm periods) are generated using the same statistical attributes for the Coshocton climate. Mainly, the runoff is most affected by the different patterns of rainfall intensity. Table 5 hints that runoff, in view of its variability for different rainfall durations, is difficult to identify accurately for the particular timescale (hourly) used in the water budget simulations. However, the other components comprising the water budget partitioning do not differ much for this site when the rainfall duration sequence is changed.

4.1. Identifying Initial Water Content

A precise initial value for profile saturation may be of minor importance because it does not influence long-term predictions. As an example, using the initial saturation of 0.72 instead of the value 0.25 (the base case value), it was found that the profile saturation converged to the base case curve in about 100 days of the first year. The higher initial saturation had a minor influence on the long-term water balance at this site.

4.2. Identifying the Profile Depth

The relationship between runoff and drainage indicated in Figure 6 was derived for a 150-cm profile depth, although the lysimeters are actually 244 cm deep. As noted earlier, the soils were variable over the depth of the lysimeters. Using $K_s = 0.64$ cm/h and a soil profile depth of 244 cm, it was found that $S_f = 0.353$ produced a water budget consistent with the measured drainage (other parameters were at the base case values). Runoff was nearly unchanged under these conditions, as would be expected when runoff is determined primarily by the saturated hydraulic conductivity. Therefore increasing the profile depth requires increasing the S_f value to achieve about the same water budget components, all other parameters being the same. Thus the appropriate S_f for a site will depend on the presumed profile depth.

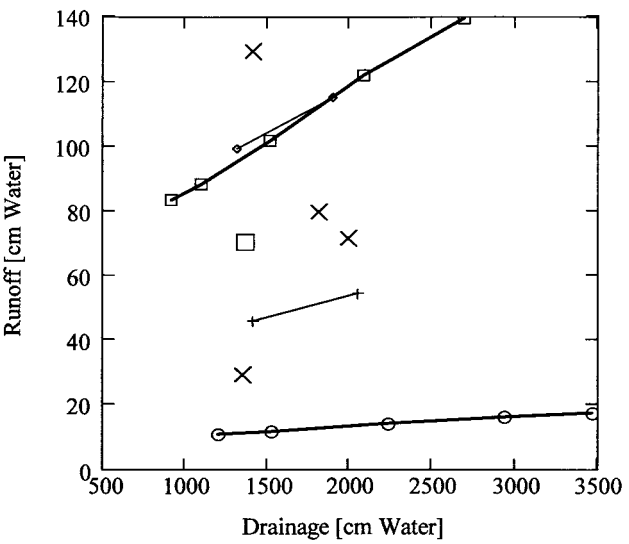

Figure 6. Runoff and drainage for four lysimeters at Coshocton, Ohio. Larger square is the budget model prediction with $K_s = 0.64$ cm/h, $m = 0.653$, $\psi_s = -35.3$ cm, $\theta_s = 0.43$, and $S_f = 0.233$. Crosses indicate the lysimeter measurements, and the other symbols (smaller squares, circles, diamonds, and plus signs) are based on parameters as indicated in Figure 4.

4.3. Importance of Redistribution

The assumption that the profile is relatively shallow and that redistribution can be treated as instantaneous may have an important impact on the accuracy of the predicted water budget. A numerical solution of Richards' unsaturated flow equation, using the same ET model as a sink and accounting for water redistribution in the unsaturated profile described by the parameters of Table 1, yielded nearly the same results obtained with the simplified model. The numerical method of *Celia et al*. [1990] was used to solve Richards' equation for a discretized profile under the same boundary conditions for the climate and drainage. However, the ET sink term was determined by the saturation distribution as is present in each discretized element of the profile (as caused by water redistribution). Thus the ET sink then accounted for the profile distribution of saturation and did not depend on only the profile average as imposed in the simplified model.

Limiting the model comparison to the study period's first half, 25 years, the averages of yearly percent relative differences in saturation, ET, and drainage were 14.5, 2.5, and 8%, respectively (i.e., the averages of absolute differences in yearly predictions divided by each yearly value from the simplified model, in percent). The greatest percent difference occurs for the saturation, with the simplified model's prediction being consistently less than that given by the profile-average saturation obtained from the Richards' equation solution. Yearly predictions by the simplified model for ET were usually greater than, and the drainage was usually less than, that given by the more exact numerical solution. Thus, for the Coshocton profile parameters, the assumption that the detailed redistribution of water need not be simulated was reasonable. Generally, such a comparison should be done to justify using the simplified model with its imposed condition of instantaneous water redistribution.

5. Summary and Conclusions

Although not entirely conceptually accurate, the simplified water budget model presented here offers an efficient, rapid approach for estimating the water budget over long periods, without having to model the detailed unsaturated flow dynamics in a profile. Such a model is ideal for doing Monte Carlo simulations of the areal variation in drainage caused by spatially variable hydraulic properties or rainfall input [*Kim et al*., 1997]. The water budget model presented here allows more flexibility for describing infiltration and evapotranspiration than the model of *Kim et al*. [1996] on which it is based. This flexibility is achieved, without increasing the computational requirements, by introducing a single additional parameter into the evapotranspiration model, the falling value of saturation. This parameter sets the minimum saturation at which the evapotranspiration rate is at its maximum. Below this point, the evapotranspiration rate is proportional to the soil profile saturation. With the more flexible model, it was possible to make the prediction of cumulative drainage conform to lysimeter measurements at the site near Coshocton, Ohio. An alternative infiltration model was also included in the water budget model that should be more accurate for extended rainfall durations.

The model's conceptual simplification of having uniform hydraulic properties over the profile root zone is reflected in an unusual result: The appropriate effective saturated hydraulic conductivity can be identified by matching the expected runoff for the particular long-term climate of the example site. On the other hand, because runoff is determined by the hydraulic conductivity for the entire profile, instead of by the surface soil conditions, there is a definite conceptual limitation on the model's accuracy. When the predicted runoff is inconsistent with the water budget of a particular site and provided it is a small part of the water balance, then it still may be reasonable to use the simplified model to estimate drainage and evapotranspiration. It was also found that the falling value of saturation ultimately determines the site-specific overall drainage, as well as the appropriate associated actual cumulative evapotranspiration. By matching the measured runoff and drainage for a site using the water budget model, it is possible to produce an estimate of evapotranspiration that is consistent with the site climate. Such matching can be accomplished rapidly as a computational advantage afforded by the model's mathematical simplifications.

The validity of the implemented simplifications applied to a particular site, however, should always be evaluated with a more detailed water budget simulator, such as that based on Richards' unsaturated flow equation. In particular, the correctness of whether plant roots can extract water as transpiration uniformly from the entire profile must be evaluated carefully when simulating a long-term water budget. In many cases, the simplification of imposing water extraction by roots reaching a constant depth may be unreasonably inaccurate for simulating a long-term water budget. Moreover, it is important to examine the model's accuracy when imposing instantaneous water redistribution following rainfall infiltration events. Thus it is necessary to apply the simplified water budget model with all due caution and with consideration of its inherent conceptual limitations.

Acknowledgments. This work was supported by the U.S. Nuclear Regulatory Commission, Office of Nuclear Regulatory Research, under project JCN W6933.

References

Bonta, J. V., and M. Muller, Evaluation of the Glugla method for estimating evapotranspiration and groundwater recharge, *Hydrol. Sci. J*., *44*(5), 743–761, 1999.

Broadbridge, P., and I. White, Constant rate rainfall infiltration: A versatile nonlinear model, 1, Analytic solution, *Water Resour. Res*., *24*(1), 145–154, 1988.

Brooks, R. H., and A. T. Corey, Hydraulic properties of porous media, *Hydrol. Pap. 3*, Colo. State Univ., Fort Collins, 1964.

Celia, M. A., E. T. Bouloutas, and R. L. Zarba, A general mass-conservative numerical solution for the unsaturated flow equation, *Water Resour. Res*., *26*(7), 1483–1496, 1990.

Chen, C., Rainfall intensity-duration-frequency formulas, *J. Hydraul. Eng*., *109*(12), 1603–1621, 1983.

Fayer, M. J., M. L. Rockhold, and M. D. Campbell, Hydrologic modeling of protective barriers: Comparison of field data and simulation results, *Soil Sci. Soc. Am. J*., *56*(3), 690–700, 1992.

Feddes, R. A., P. J. Kowalik, K. K. Malinka, and H. Zaradny, Simulation of field water uptake by plants using a soil water dependent root extraction function, *J. Hydrol*., *31*, 13–26, 1976.

Harrold, L. L., and F. R. Dreibelbis, Evaluation of agricultural hydrology by monolith lysimeters, 1944–1955, *Tech. Bull. 1179*, U.S. Dep. of Agric., Washington, D. C., 1958.

Harrold, L. L., and F. R. Dreibelbis, Evaluation of agricultural hydrology by monolith lysimeters, 1956–1962, *Tech. Bull. 1367*, U.S. Dep. of Agric., Washington, D. C., 1967.

Kelley, G. E., W. M. Edwards, L. L. Harrold, and J. L. McGuinness, Soils of the North Appalachian Experimental Watershed, *Misc. Publ. 1296*, U.S. Dep. of Agric., Washington, D. C., 1975.

Kim, C. P., and J. N. M. Stricker, Influence of spatially variable soil

hydraulic properties and rainfall intensity on the water budget, *Water Resour. Res.*, *32*(6), 1699–1712, 1996.

Kim, C. P., J. N. M. Stricker, and P. J. J. F. Torfs, An analytical framework for the water budget of the unsaturated zone, *Water Resour. Res.*, *32*(12), 3475–3484, 1996.

Kim, C. P., J. N. M. Stricker, and R. A. Feddes, Impact of soil heterogeneity on the water budget of the unsaturated zone, *Water Resour. Res.*, *33*(5), 991–999, 1997.

Marani, M., G. Grossi, F. Napolitano, M. Wallace, and D. Entekhabi, Forcing, intermittency, and land surface hydrologic partitioning, *Water Resour. Res.*, *33*(1), 167–175, 1997.

Meyer, P. D., M. L. Rockhold, W. E. Nichols, and G. W. Gee, Hydrologic evaluation methodology for estimating water movement through the unsaturated zone at commercial low-level radioactive waste disposal sites, *NUREG/CR-6346*, U.S. Nucl. Regul. Comm., Washington, D. C., 1996.

Mustonen, S. E., and J. L. McGuinness, Lysimeter and watershed evapotranspiration, *Water Resour. Res.*, *3*(4), 989–996, 1967.

Parlange, J.-Y., W. L. Hogarth, D. A. Barry, T. S. Steenhuis, M. B. Parlange, R. Haverkamp, P. J. Ross, and L. Li, Comment on "Series solution for Richards equation under concentration boundary conditions and uniform initial conditions" by Guido Daniel Salvucci, *Water Resour. Res.*, *34*(11), 3211, 1998.

Philip, J. R., The theory of infiltration, 4, Sorptivity and algebraic infiltration equations, *Soil Sci.*, *84*, 257–264, 1957.

Salvucci, G. D., Series solution for Richards equation under concentration boundary conditions and uniform initial conditions, *Water Resour. Res.*, *32*(8), 2401–2407, 1996.

Salvucci, G. D., Reply, *Water Resour. Res.*, *34*(11), 3213, 1998.

Salvucci, G. D., and D. Entekhabi, Equivalent steady state soil moisture profile and the time compression approximation in water balance modeling, *Water Resour. Res.*, *30*(10), 2737–2749, 1994.

Sander, G. C., I. F. Cunning, W. L. Hogarth, and J.-Y. Parlange, Exact solution for nonlinear, nonhysteretic redistribution in vertical soil of finite depth, *Water Resour. Res.*, *27*(7), 1529–1536, 1991.

Shuttleworth, W. J., Evaporation, in *Handbook of Hydrology*, edited by D. R. Maidment, pp. 4.1–4.53, McGraw-Hill, New York, 1993.

Smith, R. E., C. Corradini, and F. Melone, Modeling infiltration for multistorm runoff events, *Water Resour. Res.*, *29*(1), 133–144, 1993.

Streile, G. P., K. D. Shields, J. L. Stroh, L. M. Bagaasen, G. Whelan, J. P. McDonald, J. G. Droppo, and J. W. Buck, The Multimedia Environmental Pollutant Assessment System (MEPAS): Source-term release formulations, *PNNL-11248*, Pacific Northwest Natl. Lab., Richland, Wash., 1996.

Warrick, A. W., D. O. Lomen, and A. Islas, An analytical solution to Richards' equation for a draining soil profile, *Water Resour. Res.*, *26*(2), 253–258, 1990.

Warrick, A. W., A. Islas, and D. O. Lomen, An analytical solution to Richards' equation for time-varying infiltration, *Water Resour. Res.*, *27*(5), 763–766, 1991.

P. D. Meyer and C. S. Simmons, Environment Technology Division, Pacific Northwest National Laboratory, Battelle Boulevard, P.O. Box 999, Richland, WA 99352. (cs_simmons@pnl.gov)

(Received February 7, 2000; revised June 26, 2000; accepted June 30, 2000.)